

# The Semigroups Package

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# Outline of this talk

- ① A discussion of computational mathematics.
- ② (Computational) semigroup theory.
- ③ What is the Semigroups package?

# Examples of mathematical software

- Maple
- Mathematica
- R
- GAP
- MATLAB
- MAGMA

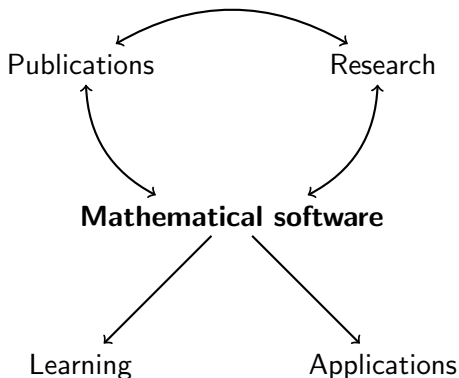


# The uses of mathematical software

- Mathematical research:
  - Finding **proofs** for theorems,
  - Constructing **computer-examples** to conjectures,
  - Spotting **patterns** in data,
  - Experimentation.
- Teaching and learning.
- Accessing data libraries.
- Performing calculations (real world or theoretical).

# What is computational mathematics?

- The **use** of computer tools to extend mathematical knowledge.
- The **development** of mathematical software.
- The mathematical **research** which enables these tools.



# Semigroup theory

A semigroup is

- ① a **set** of elements, with
- ② an **associative** way of combining two elements.

**Example:** all  $2 \times 2$  matrices over  $\mathbb{F}_2$  with matrix multiplication.

- Semigroup theory is a type of **abstract algebra**.
- Semigroup theory generalises **group theory**.

# Computational semigroup theory

Algebra

Computational algebra

Group theory & **Comp. maths** = Computational group theory

Semigroup theory

Computational semigroup theory

# Computational semigroup theory

## Centre for Interdisciplinary Research in Computational Algebra

Algebra

Computational algebra

Group theory & **Comp. maths** = Computational group theory

Semigroup theory

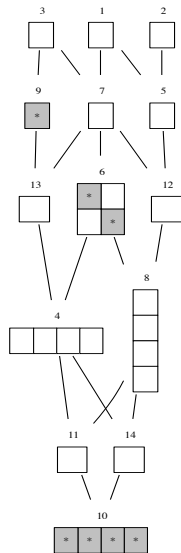
Computational semigroup theory

**Semigroups package for GAP**

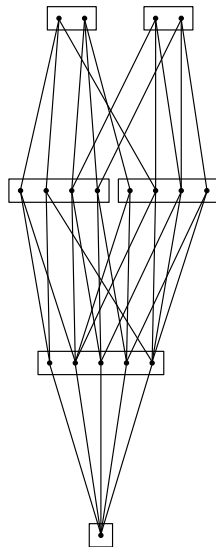
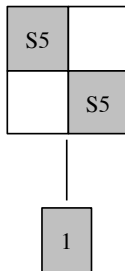




# Features: Diagrams of a semigroup



- Information at a glance.
- Impractical to make by hand.



## Features: Visualising elements of a semigroup

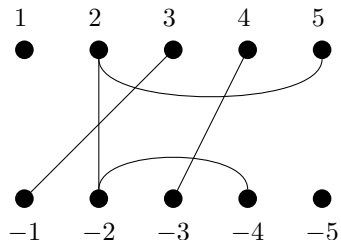


Figure: A bipartition of degree 5.

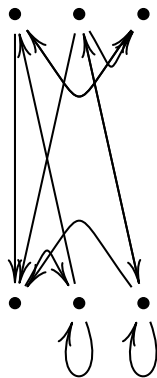


Figure: A partitioned binary relation.

# Who are we?

The Semigroups package is led by **James Mitchell**.

Current/recent contributors at St Andrews include:

- Fernando Flores Brito,
- Markus Pfeiffer,
- Julius Jonušas,
- Chris Russell,
- Michael Torpey,
- Wilf Wilson.

Many others have contributed.

## What do we do? Strategies

- Traditional pure maths research in semigroup theory.
- Implement our own new algorithms.
- Implement theoretical algorithms from the literature.
- Adapt algorithms from group theory.
- Extend or improve existing algorithms.

We **research**, we write **papers**, **software**, and **documentation**.

## What do we do? Some current and recent projects

- Congruences on finite and infinite semigroups.
- Computing maximal subsemigroups.
- Matrices over a finite field.
- E-unitary inverse semigroups.

**Semigroups 3.0 is in development.**

# Development model

## Semigroups

Download .tar.gz

View On GitHub

Version 2.8.0

This project is maintained by  
J.D. Mitchell

## GAP Package Semigroups

The **Semigroups** package is a **GAP** package containing methods for semigroups, monoids, and inverse semigroups, principally of transformations, partial permutations, bipartitions, subsemigroups of regular Rees 0-matrix semigroups, free inverse semigroups, free bands, and semigroups of matrices over finite fields.

**Semigroups** contains more efficient methods than those available in the **GAP** library (and in many cases more efficient than any other software) for creating semigroups, monoids, and inverse semigroup, calculating their Green's structure, ideals, size, elements, group of units, small generating sets, testing membership, finding the inverses of a regular element, factorizing elements over the generators, and many more. It is also possible to test if a semigroup satisfies a particular property, such as if it is regular, simple, inverse, completely regular, and a variety of further properties.

There are methods for finding congruences of certain types of semigroups, the normalizer of a semigroup in a permutation group, the maximal subsemigroups of a finite semigroup, and smaller degree partial permutation representations of inverse semigroups. There are functions for producing pictures of the Green's structure of a semigroup, and for drawing bipartitions.

The current version of this package is version 2.8.0. For more information, please refer to the [package manual](#). There is also a [README.md](#) and a [CHANGELOG.md](#) file.

- Free open-source software.
- Hosted on GitHub:
  - Issue tracker.
  - Pull requests.

**We meet every Wednesday afternoon to work on the Semigroups (and Digraphs) packages.**

